Stability Analysis of q-axis Rotor Flux Based Model Reference Adaptive System Updating Rotor Time Constant in Induction Motor Drives

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(Invited)

Abstract—q-axis rotor flux can be chosen to form a model reference adaptive system (MRAS) updating rotor time constant online in induction motor drives. This paper presents a stability analysis of such a system with Popov’s hyperstability concept and small-signal linearization technique. At first, the stability of q-axis rotor flux based MRAS is proven with Popov’s Hyperstability theory. Then, to find out the guidelines for optimally designing the coefficients in the PI controller, acting as the adaption mechanism in the MRAS, small-signal model of the estimation system is developed. The obtained linearization model not only allows the stability to be verified further through Routh criterion, but also reveals the distribution of the characteristic roots, which leads to the clue to optimal PI gains. The theoretical analysis and the resultant design guidelines of the adaptation PI gains are verified through simulation and experiments.

Index Terms—Design of the adaptation PI gains, rotor time constant, stability analysis, small-signal model.

I. INTRODUCTION

OTOR time constant, the ratio of rotor inductance to resistance, is crucial to accuracy of field orientation in indirect vector controlled induction motor (IM) drives [1-3]. In applications, rotor resistance always undergoes considerable variation due to the ohmic heating, while rotor inductance varies with the field saturation [4, 5]. Therefore, online estimation of rotor time constant is essential for achieving the full advantage of indirect field orientation control method in high-performance IM drives.

Online updating of rotor time constant has been researched for decades and many estimation schemes have been proposed [2], [6]-[16]. Model reference adaptive system (MRAS) based scheme is attractive for its simple structure and easy implementation [6]. Depending on the functional candidates chosen in the MRAS, e.g., rotor flux [6], stator voltage [7, 8], or reactive power [9, 10], various methods are available. In [6], it was pointed out that different MRAS based algorithms have its distinct advantages and limitations and some one of them may be suitable mostly to a specific application. As well known, q-axis rotor flux is ideally zero under steady operation when there is no detuning in the rotor time constant, which yields the slip frequency. Therefore, q-axis rotor flux based MRAS is a reliable estimator of rotor time constant if the flux can be obtained accurately [6]. In order to avoid flux calculation, d-axis stator voltage can be chosen instead in the MRAS. Some practical issues about d-axis stator voltage based MRAS, such as stator resistance variation, inverter nonlinearities were analyzed and addressed in [7] and [8]. Popov’s hyperstability concept and Lyapunov theorem were utilized in [9] and [10] respectively to prove stability of the reactive power model. Selection of reactive power as the functional candidate in the MRAS automatically makes the system immune to the variation of stator resistance [9]. In [11], the transient response of stator voltages was chosen as an estimator to correct the slip gain (inverse of the rotor time constant). The small-signal jitter in a MRAS-based speed estimator was wisely extracted to achieve online updating of rotor time constant. In [13], the rotor resistance and inverse of the rotor time constant were updated using the equivalent control components in the designed sliding mode observer with two manifolds. The approach based on Extended Kalman Filter was reported in [14]. The authors in [15] and [16] proposed an algorithm updating the rotor time constant and the rotor speed in parallel by superimposing extra signal on the flux command to satisfy persistent excitation requirement.

In this paper, q-axis rotor flux based MRAS updating rotor time constant online is under investigation. Although this algorithm was proposed many years ago, stability analysis of such a system is still not available up to now. Moreover, design of the gains in the proportional-integral (PI) controller, as the adaptation mechanism in the MRAS, has been receiving little attentions, although they specify the updating process and the final performance of drives. Therefore, this paper presents a comprehensive stability analysis of the system utilizing Popov’s hyperstability theory, and small-signal model technique. In Section II, the dynamic model of IM and the
q-axis rotor flux based MRAS are presented. Based on that, in Section III hyperstability concept is introduced to prove the stability of the MRAS observer. In order to further verify the stability of this q-axis rotor flux model, as well as to find out the possible guidelines for optimal design of the PI gains, a set of nonlinear equations describing mathematically the estimation system, involving the dynamics of the rotor flux, is established in section IV. Then, small-signal linearization technique is resorted to find out the characteristic equation. It is concluded that for a wide range of the PI gains the q-axis rotor flux model could work stably throughout the whole range of speed. Though, the gains can affect the converging process, which is analyzed in detail. For optimizing the PI gains, a guideline is derived by testing the root distribution of the characteristic equation. Simulation and experiments are also given, in Section V and VI, to demonstrate the effectiveness of the proposed analysis and design strategy. In Section VII, this paper is concluded.

II. IM MODEL AND Q-AXIS ROTOR FLUX BASED MRAS

A. Dynamic Model of an Induction Motor

In synchronous reference frame an IM can be mathematically described as (1)–(2) [17].

\[
\dot{u}_s = R_i + j\omega L_m \dot{L}_m + J_s \dot{\omega} L_m \dot{\omega} + L_m \dot{\psi}_r + L_m \frac{d}{dt} \psi_r, \\
\frac{d}{dt} \psi_r = \frac{L_m}{T_r} i_s - j\omega L_m \psi_r - J_s \sigma \psi_r, 
\]

where \( i_s = [i_{sd} \ i_{sq}]^T \), \( u_s = [u_{sd} \ u_{sq}]^T \), \( \psi_r = [\psi_{rd} \ \psi_{rq}]^T \), \( \sigma = 1 - \frac{L_m}{L_s L_r}, \ J_s = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \), \( T_s \) and \( T_r \) are the rotor time constant used to calculate the slip frequency and its actual value. \( \omega_k \), \( \omega_s \), and \( \omega_r \) are stator, slip, and electrical rotor angular frequencies. Stator frequency is obtained by \( \omega_k = \omega_s + \omega_r \).

B. q-axis Rotor Flux Based MRAS

Writing (1) and (2) in steady-state form and making some manipulations, one can obtain q-axis rotor flux as:

\[
\psi_{rq} = L_m i_{rq}^2 + \frac{1 - \frac{T_s}{T_r}}{\frac{T_s}{T_r}} i_{rq} \dot{i}_{rq} 
\]

It is apparent from (3) that the residue in the q-axis flux is resulted from the mismatch in the rotor time constant. Moreover, the sign of the q-axis component of stator current decides the changing direction of the q-axis rotor flux for the same mismatch in the rotor time constant, namely, the error signal input into the adaptive mechanism should be signed properly to confirm its effectiveness in the case of negative q-axis current. The block diagram of q-axis rotor flux based MRAS for online estimation of rotor time constant is shown in Fig. 1. In this paper, estimation of the rotor flux is not addressed, since it has been reported in several papers [18, 19]. The flux estimation method proposed in [18] is adopted in this paper.

The estimate of \( \psi_{rq} \) is compared with zero and the result is manipulated into a tuning signal fed into a PI controller, acting as an adaptive mechanism, to adjust the slip gain, which can be presented in a mathematical way as (4).

\[
\frac{1}{T_r} = \frac{1}{T_{ini}} + k_q k_q \psi_{rq} + \int k_q k_q \psi_{rq} dt 
\]

where \( k_q = \text{sgn}(i_{sq}) \), \( \text{sgn}() \) denotes the sign function, \( i_{sq} \) denotes q-axis stator current reference, \( T_{ini} \) denotes the initial value of rotor time constant instrumented in the controller of drive, \( k_p \) and \( k_i \) are the adaptation PI gains. Since in steady state, the sensed q-axis stator current equals its reference for a properly designed closed-loop current control, \( \text{sgn}(i_{sq}) \) is used to replace \( \text{sgn}(i_{sq}) \). Moreover, it is worth mentioning that the effects of the sign of \( i_{sq} \) on the error polarity, as found out in (3), is taken into account in this adaptive mechanism.

III. STABILITY ANALYSIS UTILIZING HYPERSTABILITY THEORY

Based on (1) and (2), the dynamic model of IM in the synchronous reference frame can also be expressed as (5) in state equation form with the stator current and the rotor flux as state variables [20].

\[
\frac{d}{dt} \begin{bmatrix} i_s \\ \psi_r \end{bmatrix} = A \begin{bmatrix} i_s \\ \psi_r \end{bmatrix} + Bu_s 
\]

where

\[
A = \begin{bmatrix} \frac{R_s + L_s^2}{\sigma L_s i_s} & \frac{L_m}{\sigma L_s L_r} \left( \frac{1}{T_s} i_s - \omega_s J_2 \right) \\ \frac{L_m}{T_r} i_s & -\frac{1}{T_r} i_s - \omega_s J_2 \end{bmatrix} 
\]

and

\[
B = \begin{bmatrix} \frac{1}{T_s} \\ \frac{1}{T_r} \end{bmatrix} 
\]
In the MRAS observer, the IM itself is regarded as the adjustable model, where the rotor time constant yielding slip frequency is unknown and remains to be estimated. Correspondingly, the reference model is presented in (6), where subscript 'ref' is used to note the reference model. The difference between the two models lies in the reference frame. (5) is referred to a synchronous reference frame oriented with the estimated rotor time constant, while (6) is referred to a synchronous reference frame oriented with actual rotor time constant.

\[
\frac{d}{dt} \begin{bmatrix} i_{\text{ref}} \\ \psi_{\text{ref}} \end{bmatrix} = A_{\text{ref}} \begin{bmatrix} i_{\text{ref}} \\ \psi_{\text{ref}} \end{bmatrix} + B_{\text{ref}}u_{\text{ref}}
\]

It is worth noting that both (5) and (6) are just used to describe this estimation algorithm equivalently in MRAS way for theoretical analysis, but neither of the two models is needed in implementation. Actually, it is not possible to calculate any of them in controller, since they all need the actual \( T_r \). Correspondingly, the slip frequency terms in \( A_{\text{ref}} \) is also different from \( A \). In \( A_{\text{ref}} \), \( \omega_s \) is the actual slip frequency of the IM under the operation state, which equals the calculated results with the used rotor time constant \( T_r \). However, in \( A_{\text{ref}} \) the slip frequency, noted as \( \omega_{\text{ref}} \), can be regarded as the desired value obtained through the actual value of the rotor time constant \( T_r \). Subtracting (6) from (5), one can get the error dynamics, as (7).

\[
\frac{de}{dt} = A_e e - W
\]

where

\[
A_e = \begin{bmatrix}
R + \frac{L_m^2}{\sigma_L} & \frac{L_m}{\sigma_L} \left( \frac{1}{T_l} - \omega_s J_r \right) \\
-\frac{L_m}{\sigma_L} \frac{1}{T_l} & \frac{1}{T_l}
\end{bmatrix}
\]

\[
e = \begin{bmatrix} i - i_{\text{ref}} \\ \psi - \psi_{\text{ref}} \end{bmatrix},
\]

\[
W = \begin{bmatrix}
u - u_{\text{ref}} - \omega_s J_r i + \omega_{\text{ref}} J_r i_{\text{ref}} \\
-\omega_s J_r \psi + \omega_{\text{ref}} J_r \psi_{\text{ref}}
\end{bmatrix}.
\]

According to (7), the equivalent nonlinear feedback system of the MRAS observer can be described in Fig. 2. Following Popov’s theorem the system in Fig. 2 is hyperstable, if the following two criterions are satisfied [9].

1) The forward-path transfer matrix is strictly positive real.

\[
\int e^T W d\tau \geq -\gamma^2 \quad \forall \tau > 0
\]

2) The nonlinear feedback block satisfies Popov's criterion of (8).

\[
e^T W = \left( \frac{1}{T_e} - \frac{1}{T_r} \right) L_m i_d \psi_q
\]

According to the following well-known inequality (10), the adaptive scheme, shown in (11), is sufficient to satisfy the second criterion.

\[
\int k \frac{df(r)}{dr} f(r) dr \geq -\frac{1}{2} k f(0)^2 \quad \forall k > 0
\]

\[
\frac{d}{dr} \frac{1}{T_r} = L_m i_d \psi_q
\]

In literature, PI controller is usually used in application, as presented in (4). Comparing (4) and (11), it can be concluded that the stability of \( q \)-axis rotor flux model is proved by Popov’s hyperstability theory. Besides, it should be noted out that the amplitude of \( q \)-axis stator current in (11) can be regarded as having been incorporated into the PI gains in (4).

IV. SMALL-SIGNAL STABILITY ANALYSIS

A. Small-Signal Model

Neglecting the stator current dynamics, the overall system, involving the rotor flux dynamics and the updating dynamics, can be described with (12), which is obtained by making some manipulations on (2) and (4).

\[
\frac{d}{dr} x = f(x)
\]

where

\[
x = \begin{bmatrix} 1/T_r \psi_d \psi_q \end{bmatrix}, f(x) = \left[ f_1(x) f_2(x) f_3(x) \right]^T
\]

and

\[
f_1(x) = k_p k_q \left( \frac{L_m}{T_r} i_d - \frac{1}{T_r} \psi_q \right) + k_q \psi_q
\]

\[
f_2(x) = \frac{L_m}{T_r} i_d - \frac{1}{T_r} \psi_q + \omega \psi_q
\]

\[
f_3(x) = \frac{L_m}{T_r} i_d - \omega \psi_q - \frac{1}{T_r} \psi_q
\]

Linearizing the nonlinear system around the desired equilibrium point \( x=x^* \), one can get:

\[
\frac{dx}{dt} = C(x-x^*)
\]

where
According to (15), the characteristic equation can be obtained as (16).

\[ s^3 + c_2 s^2 + c_1 s + c_0 \]

where

\[ c_2 = \frac{2}{T_i^2} + k_p k_q L_m i_{sq} \]
\[ c_1 = \frac{1}{T_i^2} \left( 1 + \frac{i_{sq}^2}{i_{sq}} \right) + k_q L_m i_{sq} \left( k_p \frac{1}{T_i} + k_i \right) \]
\[ c_0 = L_m i_q k \frac{1}{T_i} k_i \]

To find out the stability of the system, the Routh Table is given as:

<table>
<thead>
<tr>
<th>s^3</th>
<th>1</th>
<th>c_1</th>
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</thead>
<tbody>
<tr>
<td>s^2</td>
<td>c_2</td>
<td>c_0</td>
</tr>
<tr>
<td>s^1</td>
<td>c_1 - c_0/c_2</td>
<td>0</td>
</tr>
<tr>
<td>s^0</td>
<td>c_0</td>
<td>0</td>
</tr>
</tbody>
</table>

Obviously, the necessary conditions for stability of the parameter estimation system can be obtained as:

\[ \frac{2}{T_i^2} + k_p k_q L_m i_{sq} > 0 \]
\[ c_2 \left( \frac{1}{T_i^2} \left( 1 + \frac{i_{sq}^2}{i_{sq}} \right) + c_0 \frac{k_p}{k_i} + c_0 \frac{k_i c_q^2 T_i^2}{k_q} \right) > 0 \]
\[ L_m i_q k \frac{1}{T_i} k_i > 0 \]

As explained in (4), \( k_q \) notes the sign of the \( q \)-axis stator current, therefore, the multiplication of \( k_q i_{sq} \) is positive, which means the first and third inequality in (18) is naturally satisfied. Besides, it can be found that the second inequality in (18) is also satisfied. Therefore, all of the conditions in (18) are real, which independent of the operation speed and the specific magnitudes of the PI coefficients. Therefore, it can be definitely concluded that the \( q \)-axis rotor flux model could work stably in whole range of rotor speed and a relatively wide range of adaptation PI gains.

**B. Design of Adaptation PI Gains**

Although as having been verified through Popov’s theorem the stability of the whole IFOC system is independent of the PI gains in theory, the adaptation PI gains would specify the updating process. Using (16), the design guidelines for the adaptation PI gains will be discussed by testing the root distribution of the characteristic equation. In the case of \( k_p = 0 \), the poles of (16) for three different integral gains are pointed in the map, as shown in Fig. 3. It is apparent that there are one real and a pair of conjugate complex poles in the system. Moreover, it can also be observed that with the increase of \( k_i \), the real pole will move to left and the complex poles will move to right, resulting in reduction of the system damping. Therefore, it can be concluded that in a certain range a higher \( k_i \) will accelerate the convergence process, and as a tradeoff it will introduce unsatisfied underdamping oscillation in the estimated rotor time constant. From Fig. 3, the integral gain is selected as \( k_i = 50 \) for the tested IM. In the case of \( k_i = 50 \), three sets of poles corresponding to three proportional gains are presented in Fig. 4. It can be seen that the real pole has the potential to move toward right with the increase of \( k_p \), though the moving distance is very limited. Therefore, the gain \( k_p \) has not noticeable effect on the convergence rate. Based on (14) and linearization analysis tool in Matlab/Simulink, Bode plot of the transfer function between estimated slip gain and its actual value with different adaptation gains is given in Fig. 5. As shown in Fig. 5(b), it is apparent that increased \( k_p \) would influence the amplification of the noise in the estimated rotor time constant. This conclusion is confirmed by the experimental results shown in Fig. 12. Based on the above analysis, a pure integral controller is adopted in the MRAS and the integral gain is chosen as \( k_i = 50 \).
In order to validate the proposed theory and design strategy, an induction motor drive under IFOC is mathematically modeled in MATLAB/Simulink. The machine parameters are given in Table I in the Appendix. In the simulation, the estimated rotor time constant yielding the slip is changeable arbitrarily while the rotor time constant of the motor is presumed to be constant. At beginning, two incorrect estimated slip gain, namely $1/T_\text{r} = 2s^{-1}$ and $8s^{-1}$ is set as the initial values respectively. At 0s, the motor has been operated in steady-state. At 2s, the updating algorithm is activated.

Fig. 6 presents the simulation results of the updating algorithm with different integral gains in the adaptation PI controller, where the $q$-axis current is set to 40.8A and motor is operated at 1200r/min. It can be seen that higher $k_i$ increases the convergence rate. However, the oscillation appears in estimate of the slip gain due to the underdamped characteristic, which is consistent with the theoretical results. Fig. 7 presents the simulation results of the algorithm with different proportional gains under the condition $k_i = 50$. Comparing Fig. 7(a) and (b) against Fig. 6(b), it can be found that the proportional gain results in a large impulse in the estimate, which is not beneficial to the system stability.

In Fig. 8, the performance of $q$-axis rotor flux model drive is tested for an extreme case, i.e., step change in initial value of slip gain [9]. Fig. 9 gives the simulation results of rotor time constant at a low speed of 30 r/min and a high speed of 2400 r/min. It is clearly shown that the estimated slip gain still can converge to its actual value under different operation speeds.
VI. EXPERIMENTAL RESULTS

The proposed analysis are further verified on an experimental platform with Texas Instruments digital signal processor, TMS320F28335. As shown in Fig. 10, the experimental platform consists of a 10-kW induction motor operating in the torque mode, and a 22-kW induction motor serving as the mover to maintain the speed of the machine set. The parameters of the tested motor are the same with those used in the simulation. In the experiments, the actual value of rotor time constant is presumed to be constant for a short time experiment due to large thermal constant. The initial values of the estimated rotor time constant, used for slip frequency calculation, is changed deliberately.

At first, motor speed is kept at 1200r/min and \( q \)-axis current reference is set to 40.8A. When proportional gain is set to zero in the MRAS, the experimental results of the \( q \)-axis rotor flux model with different integral gains are shown in Fig. 11. Making comparison between Fig. 6 and 11, it is apparent that they are excellently correlate. Fig. 12 demonstrates the experimental results of the estimation scheme with different proportional gains. Comparing Fig. 12(a), (b) against Fig. 11(b) shows that an increased \( k_p \) not only cannot improve convergence rate, but also introduces noise into the estimate. The phenomenon is consistent with the theoretical analysis and simulation results, as shown in Fig. 5, Fig. 6 and Fig. 7, respectively.

Online updating of slip gain with a step change in initial value of slip gain is demonstrated in Fig. 13. With the motor running at low speed of 30 r/min and high speed of 2400r/min and \( q \)-axis stator current is set to 40.8A, the updating process is described in Fig. 14. It can be seen that the slip gain can always be tuned from the initial error value to its actual value.
YANG et al.: STABILITY ANALYSIS OF Q-AXIS ROTOR FLUX BASED MODEL REFERENCE ADAPTIVE SYSTEM UPDATING ROTOR TIME CONSTANT IN INDUCTION MOTOR DRIVES

Fig. 12. Experimental results for online updating of slip gain, $\nu_{eq}$ and stator current response curves with different proportional gains under $k_p=50$: (a) $k_p = 10$, (b) $k_p = 50$.

Fig. 13. Experimental results for online updating of slip gain, $\nu_{eq}$ response curves with step change in initial value of slip gain.

Fig. 14. Experimental results for online updating of slip gain, $\nu_{eq}$ and stator current response curves with different speeds for $k_p=0$, $k_i=50$: (a) 30r/min, (b) 2400r/min.

VII. CONCLUSIONS

The MRAS for online rotor time constant estimation utilizing $q$-axis rotor flux is proved to be stable over the whole speed range with accurate flux estimation. The effect of the adaptation gains in the MRAS on the updating process is given. It is shown that increased proportional gain would introduce noise in the estimated rotor time constant and a tradeoff between convergence rate and underdamp oscillation is required when selecting integral gain. A pure integral controller in $q$-axis rotor flux model is suggested. Simulation and experimental results confirm the validity of the proposed analysis and design strategy.

APPENDIX

TABLE I
PARAMETERS OF THE TESTED IM

<table>
<thead>
<tr>
<th>Rated parameters</th>
<th>values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power</td>
<td>10kW</td>
</tr>
<tr>
<td>Rated voltage</td>
<td>220V</td>
</tr>
<tr>
<td>Rated current</td>
<td>32A</td>
</tr>
<tr>
<td>Rated frequency</td>
<td>100Hz</td>
</tr>
<tr>
<td>Pole pairs</td>
<td>2</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>0.076Ω</td>
</tr>
<tr>
<td>Rotor resistance</td>
<td>0.055Ω</td>
</tr>
<tr>
<td>Magnetizing Inductance</td>
<td>13.6mH</td>
</tr>
<tr>
<td>Stator/ Rotor Inductance</td>
<td>14.1mH</td>
</tr>
</tbody>
</table>
REFERENCES


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