Abstract—In order to accurately analyze the magnetic field of conical-rotor permanent magnet synchronous motor (CR-PMSM), the effectiveness of two methods was studied on handling of problems concerned with non-uniform distribution of magnetic field along axial direction in CR-PMSM, which were sectional calculation (SC) method and three-dimensional finite element (3-D FE) method. On this basis, the influence of the axial displacement and dq-axis currents on the operating characteristics of axial magnetic force and torque is analyzed by using the 3-D FE model. Analysis results show that the axial magnetic force and torque decrease with the increase of axial displacement of the rotor, and the amplitude regularity of the axial magnetic force is affected by the d-axis current. A prototype machine is fabricated and tested, in order to validate the design theory.

Index Terms—Magnetic field of conical-rotor permanent magnet synchronous motor, operating characteristics, non-uniform of magnetic field, 3-D FE method.

I. INTRODUCTION

TRADITIONALLY, the stator and rotor core are conical. The conical-rotor has the axial movement to change the air gap length, so the inherent characteristic of self regulating excitation can be realized, the no-load back electromotive force (EMF), axial magnetic force, inductances and torque performance parameters of the motor can be changed easily. So, the conical-rotor machines are mainly used in high speed turbine power generation, and can also be used for electric vehicle driving. On the other hand, the mechanical loss, the balancing device (such as bearing, spring, etc.) and the working characteristics of fast start stop are directly affected by the axial magnetic force. Therefore, the accurate calculation of axial magnetic force is particularly important for the design and application of CR-PMSM. The axial magnetic force will not only be affected by the change of air gap length, but also the dq-axis currents of armature. Then, CR-PMSMs have attracted great interest of researchers in recent years.

For example, in [1], it presents a novel structure of PMSM with a conical-rotor and studies its flux weakening performance in detail. The CR can be moved outside in axial direction, which decreases the air-gap flux density as well as the effective space between the stator and the rotor. This method is effective in increasing the maximum speed and the output power of PMSM, However, axial magnetic force has not been analyzed.

In [2], a direct-drive conical-rotor PM synchronous generator (CR-PMSG) for turbo-expander was developed, which features its adaptive equilibrium of axial force and simple, low-cost bearing system. The relationship curve about generator rotor axial magnetic force and air gap and cone angle is obtained, obtaining the axial magnetic force under the best air gap and cone angle values, which effectively balance the turbo-expander axial force. The analysis and simulation results show that the direct-drive turbo-expander with CR-PMSG has good rotor running stability. However, axial magnetic force has not been tested.

In [3], the variation of direct axis and quadrature axis inductances were obtained by 3D finite element method for the CR-PMSM with the taper rotor at different axial locations. And the inductances of a prototype were measured at different test frequency and with taper rotor at different axial locations. However, axial magnetic force is not analyzed.

In [4], a novel type of bearingless PMSM of consists of two conical air gap bearingless PM half-motors, mounted on a single shaft was presented. The value of the axial force depends, besides of the motor geometry, on the PM air gap flux density, on the air gap cone angle and on the d-axis current. However, the effect of q-axis currents on axial force is not analyzed.

In [5], a method to control power of a synchronous generator driven by a powerful variable speed wind turbine was developed. The generator rotor has a conical shape and allows the air-gap variation through the lateral movement of the rotor subassembly, using a worm gear. During the operation, the generator is not disconnected from the grid, even at very high wind speeds because due the automatic increase of air gap the power of the generator is limited at a specific value. However, conical-rotor synchronous generator has some disadvantages, such as the complexity of technological manufacturing and mechanical assembly, and the difficulty of rotor control during operation because of the longitudinal movement.
The accurate distribution of its magnetic field is the precondition for analyzing the working characteristics of the motor. In this paper, aiming at the uneven distribution of magnetic field along the axial direction of a CR-PMSM [6], [7], the effectiveness of SC method and 3-D FE method [8] are compared to deal with the uneven distribution of axial magnetic field. The change regulation of magnetic field distribution and the no-load back EMF of the CR-PMSM is obtained and verified by experiment. The axial magnetic force and torque characteristics of the conical-rotor are analyzed under the axial displacement and dq-axis currents by using the 3-D FE model. The axial magnetic force and torque decrease with the increase of the rotor axial displacement, and the variation of axial force amplitude is affected by the d-axis current. Finally, a prototype CR-PMSM is fabricated and tested to verify the analysis of the magnetic field and motor performance, which laid a foundation for the application of this type of motor.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>DIMENSIONS OF PROTOTYPE CR-PMSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Factor</td>
<td>Value</td>
</tr>
<tr>
<td>Rated power $P_N$ (kW)</td>
<td>2.0</td>
</tr>
<tr>
<td>Rated speed $n$ (r/min)</td>
<td>6000</td>
</tr>
<tr>
<td>Cone angle $\alpha$ (°)</td>
<td>6</td>
</tr>
<tr>
<td>Core length $L_{ef}$ (mm)</td>
<td>52</td>
</tr>
<tr>
<td>Outer diameter of stator $D_i$ (mm)</td>
<td>167</td>
</tr>
<tr>
<td>Average inner radius of rotor $R_{av}$ (mm)</td>
<td>96.4</td>
</tr>
<tr>
<td>Air-gap $\delta$ (mm)</td>
<td>0.7</td>
</tr>
<tr>
<td>Stator slot number $Q_s$</td>
<td>24</td>
</tr>
<tr>
<td>Maximum axial displacement of rotor $\Delta z$ (mm)</td>
<td>4</td>
</tr>
</tbody>
</table>

II. MAGNETIC FIELD ANALYSIS OF CR-PMSM

A. Structure of CR-PMSM

The structures of CR-PMSM are compared in Fig. 1, and its main design parameters are listed in Table I.

B. Magnetic Field Distribution

Generally, for the situation of the uneven axial distribution of motor magnetic field along axial direction, SC method is mostly used to analyze [9]-[11], the motor is divided into several sections along the axial direction in this method. The length of each section should be reasonably selected to ensure that the calculation time for each section is about 1 hour. It is considered that the magnetic field is constant in each section, so the 3-D magnetic field can be simplified to the analysis of multiple two dimensional (2-D) magnetic fields, and then, the results of each small section are summed to get the motor parameters [12, 13]. In the process of SC as described above, the motor is divided into $n=10$ segments along axial direction from the smaller end of the stator diameter of the motor to the larger one, each length is only 5.2 mm. A 2-D FE model of each segment is established to calculate the magnetic field distribution of the rotor in the alignment with the stator core, as shown in Fig. 2.

The complex magnetic field in the motor can be directly calculated by 3-D FE method. Considering the symmetry of the motor, the 3-D FE model of 1/4 motor is established. The EF division and magnetic field distribution of the stator and rotor are shown in Fig. 3.

The air gap is minimal when the conical rotor is in alignment with the stator, the gap flux density ($B_\delta$) distribution at no-load is shown in Fig. 4, when SC method is used. It can be seen that the maximum gap magnetic density between the larger end of the stator diameter (10th segment) and the stator inner diameter is about 0.25T. The air gap magnetic density corresponding to the position of the stator diameters of each small section can be obtained by using 3-D FE method, as shown in Fig. 4.
Compared with the results of SC method, it is found that the air gap magnetic density is basically the same in the small outside diameter (first section) of the rotor. The air gap magnetic density is low at the larger outside diameter of the rotor (Tenth section) by 3-D EF magnetic field analysis. The magnetic density difference between the two ends of the motor is small, which is only 0.1T. In addition, the magnetic density of the radial air gap is slightly lower in the results of the 3-D FE analysis, but there is a magnetic density ($B_δz$) component along the axial (z axis), as shown in Fig. 4, the maximum value of $B_δz$ is about 0.25 T. Besides, it also can be seen that both waveforms have a 90° phase difference between each other. It is known that the magnetic field in the air gap is perpendicular to the outer surface of the rotor by analyzing the z axis and radial component of the air gap magnetic density.

Fig. 5. Profile of CR-PMSM.

When the axial displacement of a conical rotor is produced, the stator and the rotor are misplaced ($Δ_δ >0$), as shown in Fig. 5, where $Δ_δ$ indicates the air gap length of the rotor when the axial displacement.

As can be seen from Fig. 5, and the air gap can be expressed as follow:

$$Δ_δ(α, Δz) = δ_α + δz_α sin α$$  \hspace{1cm} (1)

The axial superposition magnetic field of the stator and rotor core part can be only get by using SC method, while the rotor beyond the stator part cannot be calculated, so the superposition calculation method is not applicable at this time. The 3-D FE method is also applicable to the case where the rotor core is not aligned [13], the magnetic density distribution cloud is shown in Fig. 6.

As can be seen from Fig. 6 (a), with the increase of the stagger distance between the rotor and stator, the magnetic pole surface of the rotor core has a depressed magnetic density. When the axial displacement is relatively small, the distribution of the magnetic field of the rotor core is basically the same with that of the stator, and no densification of the magnetic flux density occurs with the increase of the displacement. The flux density of the rotor is larger than that of the stator core. The end effect increases, and the magnetic dense sag begins to appear gradually. As shown in Fig. 6 (b) and (c), the depression of the magnetic pole surface of the rotor core has been obvious at the displacement of $Δz=3.0mm$. The stator magnetic field distribution is invariable, but with the increase of the axial displacement of the rotor, the stator magnetic density is different at the two ends of the motor. At the larger end of the stator bore, the magnetic density on the top of the stator tooth appears to be partially saturated, and the magnetic density on the top of the stator tooth is obviously reduced at the smaller end of the stator bore. When the axial permanent magnet width of the motor is kept constant, the magnetic density of the stator core is obviously lower because of the decrease of the magnetic flux area of the PM.

When the air gap increases, the amplitude decreases and the air gap magnetic density decreases linearly. The change of air gap magnetic density on the center line of the magnetic pole is shown in Fig. 7 as the axial displacement of the rotor increases. The maximum amplitude is reduced to 0.61 T at $Δz=4.0mm$ from 0.88 T at $Δz=0$.

Fig. 6. 3-D FE results at different rotor axial displacement.

Fig. 7. Air gap flux density of pole center at different rotor axial displacement.

### III. CHARACTERISTIC ANALYSIS OF CR-PMSM

#### A. Constant No-load Back EMF

The stator winding distribution of CR-PMSM is the same as the conventional PMSM, the same is under the effect of magnet [15]. Thus, the no-load back EMF ($E_0$) is the derivative of the flux linkage $N_φ_0$ and the speed $n$. According to (1), its can be expressed as follows:

$$E_0(n, α, Δz) = n \cdot N_δ_0 Φ_0$$

$$= n \cdot N_δ_0 \left( L_{ef, δ} - Δz \right)$$ \hspace{1cm} (2)  

where $H_m$ is thickness of the direction of the magnetization.
Obviously, with the increase of rotor axial displacement $\Delta z$ according to the motor speed $n$, the CR-PMSM can create a constant no-load back EMF, because of the decreased air-gap flux density as well as the effective length between the stator and rotor.

B. Analysis of the Axial Magnetic Force and Torque Characteristics

When the motor moves in the axial direction, the air gap length becomes larger and the coupling area of the stator and rotor decreases, thus, the magnetic field energy in the motor is changed. Change of magnetic field energy will have an impact on the axial magnetic force and torque, the general analysis methods include Maxwell stress method [16] and virtual displacement method. The virtual displacement method obtains the total value of the magnetic force by solving the change rate of the magnetic energy to the small virtual displacement, and it is more convenient as a qualitative analysis [17-19].

In this paper, the change of the torque and axial magnetic force of the rotor during axial movement is qualitatively analyzed by the virtual displacement method [20, 21], and the exact calculation is solved by numerical method. Ignoring the magnetic saturation, it is believed that the magnetic energy stored in the permanent magnet is unchanged before and after the rotor displacement, the magnetic energy in the air gap can be expressed as follows:

$$W_m = \frac{1}{2} \int B_d H dV$$  \hspace{1cm} (3)

where, $s$ is an air gap region, $H$ is field strength at air gap.

As shown in Fig. 4, the volume $dV$ of the air gap is represented as:

$$dV = \frac{R_1 + R_{1l} + \Delta \delta}{2} \cdot \Delta \delta \cdot dz \cdot d\phi$$  \hspace{1cm} (4)

where $R_1 = R_{av} + z \cdot \text{tg} \alpha$, $R_{1l} = R_1 + dz \cdot \text{tg} \alpha$, $R_1$ and $R_{1l}$ is the radius of the two ends of the volume of the air gap microelement, $\psi$ is the circular angle coordinates.

Substituting (1) and (4) into (3) yields:

$$W_m = \frac{1}{2} \int_{0}^{2\pi} \int_{\frac{\Delta \delta}{2}}^{\frac{\Delta \delta}{2}} B_d H \times \left[ R_{av} + z \cdot \text{tg} \alpha + \frac{1}{2} dz \cdot \text{tg} \alpha + \frac{1}{2} (\delta + \Delta \delta \cdot \text{tg} \alpha) \right] \times \delta + \Delta \delta \cdot \text{tg} \alpha) \ dz \cdot d\phi$$  \hspace{1cm} (5)

The axial displacement $\Delta z$ has nothing to do with the integral variable $z$, (5) can be simplified to:

$$W_m = \frac{1}{2} \lambda_1 (\Delta z)^2 \cdot \text{tg}^2 \alpha + \lambda_2 \cdot \text{tg} \alpha(\lambda_1 \delta + \lambda_2 \Delta z)$$  \hspace{1cm} (6)

where:

$$\lambda_1 = \int_{0}^{2\pi} \int_{\frac{\Delta \delta}{2}}^{\frac{\Delta \delta}{2}} B_d H dzd\phi,$$

$$\lambda_2 = \int_{0}^{2\pi} \int_{\frac{\Delta \delta}{2}}^{\frac{\Delta \delta}{2}} B_d H \times (R_{av} + z \cdot \text{tg} \alpha + \frac{1}{2} dz \cdot \text{tg} \alpha) \ dzd\phi$$

According to the principle of virtual work, the axial magnetic force and torque of the rotor can be calculated by the magnetic energy, which is $F_z = -\frac{\partial W_m}{\partial \Delta z}$, $T = -\frac{\partial W_m}{\partial \theta}$, into (6) yields:

$$F_z = \lambda_1 (\Delta z)^2 \cdot \text{tg}^2 \alpha - \lambda_2 \cdot \text{tg} \alpha(\lambda_1 \delta + \lambda_2 \Delta z)$$  \hspace{1cm} (7)

$$T = -\left[ \frac{1}{2} \frac{\partial \lambda_1}{\partial \theta} \right] \lambda_2 (\Delta z)^2 \cdot \text{tg}^2 \alpha + (\Delta z) \ tga(\frac{\partial \lambda_1}{\partial \theta} \delta + \frac{\partial \lambda_2}{\partial \theta} \Delta z)$$  \hspace{1cm} (8)

Based on the above analysis, the axial magnetic force and torque are relevant to parameters such as air gap magnetic field, air gap length, rotor outer diameter, core length, cone angle, rotor axial displacement [22, 23]. It is shown that the axial magnetic axial force of the rotor is linear with the axial displacement, when the motor dimensions are determined, and the torque may appear extreme value, the occurrence of extreme value or not depends on the coefficient in the formula (8), that is, depending on the air gap magnetic field distribution.

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>PARAMETERS OF 2.0 KW PROTOTYPE CR-PMSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
<td>Value</td>
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<tr>
<td>Power $P_n$ (kW)</td>
<td>2.0</td>
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<tr>
<td>Speed $n$ (r/min)</td>
<td>6000</td>
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<tr>
<td>Torque $T_e$ (N.m)</td>
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<td>Resistance $R$ (Ω)</td>
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<td>Pole pairs $p$</td>
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<td>Flux $\psi$ (Wb)</td>
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<td>$L_d$ (mH)</td>
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<td>$L_q$ (mH)</td>
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<td>No-load back EMF $E_0$ (V)</td>
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<tr>
<td>Axial magnetic force $F_z$ (N)</td>
<td>175</td>
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<tr>
<td>Estimated rotor inertia $J$ (kg m$^2$)</td>
<td>0.003438</td>
</tr>
<tr>
<td>Damping factor $C$ (N•m.sec/rad)</td>
<td>0.0016269</td>
</tr>
</tbody>
</table>

Fig. 8. Testing board turbo-expander direct-driven of CR-PMSM.
IV. EXPERIMENTAL VALIDATION AND ANALYSIS

A. Test Rig of Prototype Motor

In order to evaluate the performance of developed CR-PMSM, the parameters are as shown in Table II. The whole prototype test rig is shown in Fig. 8, which is composed of CR-PMSM, oscilloscope, axial displacement device, turbo-expander, volute and controller. The variation of external load will vary with the command, which shows the variation of inlet flow of valve with respect to the time.

B. Analysis of Results

The no-load back EMF of the motor is directly determined by the air gap magnetic field, so the analysis of the magnetic field of the motor can be verified by the no-load back EMF of the motor. When the stator and rotor is aligned in a CR-PMSM, the no load phase back EMF of CR-PMSM is then measured and depicted in Fig. 9, of which the RMS value is about 238 V. Compared with the 3-D FE method result, it has a quite slight difference (3%), the air gap magnetic field is large by using the SC method, and its no-load back EMF is also large, which confirms the accuracy of developed 3-D model.

Fig. 9. No-load EMF when rotor aligns with stator of CR-PMSM.

When the conical-rotor is not aligned with the stator (Δz > 0), it is known from Fig. 5 that with the increase of the axial displacement of the rotor, the air gap length increases, as shown in Formula 1, the relationship between the measured no-load back EMF and the axial displacement of the rotor is shown in Fig. 10, it can be seen that with the increase of axial displacement, the no-load back EMF of the motor basically decreases linearly.

The relationship between axial magnetic force, armature current and axial displacement is shown in Figs. 11-13.

As shown in Figs. 11, it is known that when the d-axis current is flux enhancing (id > 0), the greater the d-axis current, the stronger the air gap magnetic field, and the greater the axial magnetic pull force, when the straight axis current is demagnetization (id <0), the reverse is the case. When the q-axis current increases, the axial magnetic is slightly reduced due to the influence of the axial magnetic circuit saturation, and the axial magnetic force is symmetrical on both sides of the q-axis current. From the numerical point of view, the axial magnetic force is 175.5N when the 5A of d-axis current of flux enhancing is applied, the axial magnetic force is 152.7N, when the 5A of d-axis current of demagnetizing is applied, the axial magnetic force is 168.3N at no-load.

Fig. 11. The axial magnetic force under various d-axis currents and axial displacement of conical-rotor. (a) Calculation results under various d-axis current. (b) Experimental results under various d-axis current.

Fig. 12. The axial magnetic force under various q-axis currents and axial displacement of conical-rotor. (a) Calculation results under various q-axis current. (b) Experimental results under various q-axis current.

Fig. 13. The torque-angular characteristics at 5A of the CR-PMSM. (a) Calculation results of torque. (b) Experimental results of torque.

The axial magnetic force is 175N when the 5A q-axis current is applied, it is shown that the demagnetization current has a greater effect on axial magnetic force, and axial magnetic tensile force is symmetric on two sides of q-axis current could be seen in Fig. 12.

The relationship between the axial magnetic force and the dq-axis currents does not change after the stator and rotor are
stagger. Under the action of a certain dq-axial currents, the greater the axial displacement of the rotor, the smaller the axial magnetic force. Compared with the 3-D FE method result, it has a quite slight difference (1.5%), which confirms the accuracy of developed 3-D model.

The overall change trend of the torque-angular (β) characteristics of the prototype when the stator current (I) is 5A is shown in Fig. 13. It can be seen from the diagram that the torque decreases monotonically with the increase of the axial position of the rotor under the condition of a certain rotor angle. Compared with the 3-D FE method result, it has a quite slight difference (2%), which confirms the accuracy of developed 3-D model.

As can be seen from Figs. 11-13, both axial magnetic force and torque have been described. It is also noteworthy that some spikes/disturbances will be introduced into the results due to the limitation and discreteness of the experimental data collected.

V. CONCLUSION

In this paper, a CR-PMSM was investigated. The basic motor performances including magnetic field characteristics, no-load back EMF, axial magnetic force and electromagnetic torque, were analyzed. The distribution of the magnetic field was obtained, and based on the analysis of uneven magnetic field distribution of CR-PMSM by using 3-D FE method, the change regulations were obtained: the air gap magnetic field is linearly reduced with the increase of the axial displacement of the rotor, the d-axis current demagnetization current has a great influence on the axial magnetic force, and the torque decreases with the increase of the axial displacement of the rotor. However, it is still quite challenging to derive the expression of axial force with respect to d-axis current and cone angle, which will be the priority of the future work.

REFERENCES


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His current research interests include electrical machines design and control.
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