Vector Control of the Single-Phase Inverter Based on the Extended and Virtual Circuits

Tao Xu, Ming Yang, Naizhe Diao, Xianrui Sun, Suliang Wu, Zhuangzhuang Shen, Hongzhi Chen, and Chonghui Song

Abstract—A vector control based on the extended equivalent circuit and virtual circuits is proposed for the single-phase inverter. By the extended circuit, the other two phase voltages can be extended by the output voltage of the single-phase inverter so as to construct the voltage vector. The voltage outer-loop is to control the voltage vector in dq coordinate system, and the output voltage can track the target value without deviation in steady state. By designing the virtual circuit, the output inner-loop can achieve approximate decoupling and improve the dynamic response under the changeable load. Compared with the traditional dual closed-loop control, the proposed dual closed-loop control scheme only needs to detect and control the voltage without the current. It not only can achieve good control effect, but also reduce the complexity of the hardware. Finally, the simulation and experimental results show that the single-phase inverter has good static and dynamic characteristics regardless of stable load or changeable load.

Index Terms—Coordinate transformation, dual closed-loop control; extended circuit, single-phase inverter, vector control; virtual circuit.

I. INTRODUCTION

THAT the output voltage can quickly track the desired value is an important criterion for the single-phase inverter. A good inverter requires a good static response and fast dynamic response. Besides, it needs a hard output characteristic and a good robustness when a load changes frequently. For a single-phase inverter, the deviation based closed-loop control is widely used. The control schemes based on the traditional closed-loop control can be roughly summarized as follows: (i) the voltage and current dual closed-loop control [1-3], (ii) the voltage and current’s eigenvalue feedback control [4-5], (iii) the output feedforward on a closed-loop control [6-8], (iv) the advanced control algorithm [9-10], (v) the virtual impedance technique as an auxiliary measure of variable feedback control [11-12]. The voltage and current dual closed-loop control is difficult to eliminate steady-state error. The eigenvalue feedback control can eliminate steady-state error, but it can increase the adjustment time. The output feedforward can improve dynamic performances, whereas it makes the adjustment time increase. Although using advanced control algorithm can improve the dynamic and static performances of the inverter, it increases the complexity of the controller and the debugging difficulty. The virtual impedance technique makes performances of the inverter further improved, but it still retains the shortcomings of traditional closed-loop control.

In this paper, a vector control based on the extended equivalent circuit and virtual circuits is proposed for the single-phase inverter. By the extended circuit, the other two phase voltages can be extended by the output voltage of the inverter so as to construct the voltage vector. The voltage outer-loop is to control the voltage vector in dq coordinate system, and the output voltage can track the target value without deviation in steady state. By designing the virtual circuit, the voltage inner-loop can achieve approximate decoupling and improve the dynamic response under the changeable load. Compared with the traditional dual closed-loop control, the proposed dual closed-loop control scheme only needs to detect and control the voltage without the current. It not only can achieve good control effect, but also reduce the complexity of the hardware. Finally, the simulation and experimental results show that the single-phase inverter has good static and dynamic characteristics regardless of stable load or changeable load.

II. ANALYSIS OF THE SINGLE-PHASE INVERTER

A. Control Principle of the Single-phase Inverter

The single-phase inverter [13-15] consists of the H-bridge circuit and LC filter circuit. The circuit diagram is shown in Fig. 1. $L$ is the filter inductance and $C$ is the filter capacitor. $S_1$, $S_2$, $S_3$ and $S_4$ are Insulated Gate Bipolar Transistors, i.e., IGBT. $D_1$, $D_2$, $D_3$ and $D_4$ are freewheeling diodes. $V_{dc}$ and $u_{inPWM}$ are the DC-link voltage of inverter and the output square wave voltage of the H-bridge. $u_1$, $u_2$, and $Z_i$ are the voltage across the inductor, the output voltage of the inverter and the load impedance, respectively.

Assuming that the harmonic component of $u_{inPWM}$ can be filtered by the LC filter, the H-bridge can be equivalent to a controlled AC voltage source, i.e., $u_1$. The equivalent circuit of the single-phase inverter is shown in Fig. 2.
From the equivalent circuit, the relationship of the input voltage and output voltage is

$$u_{in} = LC \frac{d^2 u_{in}}{dt^2} + L \frac{du_0}{dt} + u_0. \quad (1)$$

The phasor relationship between these physical quantities is shown in Fig. 3.

Because of the filter inductance, $u_L$ lags behind $u_{in}$ at an angle of $\Delta \theta$. Besides, since $u_L$ is far less than $u_{in}$, $\Delta \theta$ is very small and the amplitudes of $u_L$ and $u_{in}$ are approximately equal. $\Delta \theta$ depends on the characteristic and size of the load, so $\Delta \theta$ is unknown due to unknown load. In this paper, the resistive load is taken as an example without special instructions. Thus, $\Delta \theta$ is more than zero.

The control target of the single-phase inverter is that $u_o$ is equal to the desired voltage $u_o^*$. Here, $u_o^*$ is

$$u_o^* = U^* \cos(\omega t). \quad (2)$$

From Fig. 3, when the inverter output voltage is equal to the desired voltage, the fundamental component of the input voltage should be

$$u_{in}^* = (U^* + \Delta U^*) \cos(\omega t + \Delta \theta^*) \quad (3)$$

where $\Delta U^*$ is the amplitude increment, and $\Delta \theta^*$ is the phase increment. Assume that the input voltage of the equivalent circuit is

$$u_{in} = U_{in} \cos(\omega t + \theta + \Delta \theta) \quad (4)$$

where $U_{in}$ is the magnitude of the input voltage, and $\theta + \Delta \theta$ is the initial angle. At the time, the corresponding output voltage is

$$u_o = U \cos(\omega t + \theta) \quad (5)$$

where $U$ is the magnitude of the output voltage $u_o$, and $\theta$ is the initial angle. The basic control principles of the inverter are that $U_{in}$ is equal to $U^* + \Delta U^*$ and that $\theta$ is equal to $\Delta \theta^*$. Therefore, it can be ensured that $u_o$ is entirely equal to $u_o^*$.

**B. Three-phase Extended Circuit and Construction of Space Vector**

For a closed-loop control based on the stationary coordinate system, using a typical PI regulator cannot achieve the output voltage track the desired voltage without deviation in steady state. For a closed-loop control based on the synchronous rotating coordinate system, the coordinate transformation is mainly used to transform the AC quantities of the stationary coordinate system into the DC quantities of the synchronous rotating coordinate system, so a typical PI regulator can be used to realize the output voltage track the desired voltage without deviation.

The equivalent circuit of the single-phase inverter can be seen as a one-phase loop of a three-phase inverter. In turn, the single-phase circuit can generate the other two phase so that the equivalent circuit of three-phase extension is gotten. The extended circuit is shown in Fig. 4. The three-phase voltages are $u_a$, $u_b$, and $u_c$ where $u_a = u_o$. In the three-phase extended circuit, each phase amplitudes of the three phase input voltages are equal while the phase difference is $120^\circ$. Each phase inductance, filter capacitance and load are also equal, respectively. Each phase amplitudes of the output voltages are the same and the phase difference is $120^\circ$. The $a$-phase of the three-phase extended circuit is equivalent to the equivalent circuit of the single-phase inverter. Thus, the voltage vector can be constructed by the voltages of the three-phase extended equivalent circuit.

The output voltages $u_a$, $u_b$, and $u_c$ of the three-phase extended equivalent circuit can construct the voltage vector $\vec{u}_o$. The voltage vector can be decomposed into $u_{oa}$ and $u_{ob}$ in $\alpha\beta$ coordinate system, where $u_{oa}$ is the $\alpha$-axis component and
$u_{\alpha\beta}$ is the $\beta$-axis component. Because only the single-phase voltage $u_a$ can be measured and $u_{\alpha\alpha}$, $u_a$ and $u_{\alpha\beta}$ are equal, $u_{\alpha\beta}$ can be generated by FFT. The two components synthesize the vector $\tilde{u}_o$ whose synthesis is shown in Fig. 5. Black lines represent $abc$ coordinate system, green lines represent $a\beta$ coordinate system, and the red line represents the voltage vector.

![Fig. 5. Synthetic space vector.](image)

The input equation of Fourier transformation is

$$u_o = U \cos(\omega t + \theta). \tag{6}$$

The output equation of Fourier transformation is

$$\begin{cases} 
\tilde{u}_o = u_{\alpha\alpha} + ju_{\alpha\beta} \\
u_{\alpha\alpha} = FFT(u_o)_x = U \cos(\omega t + \theta) \\
u_{\alpha\beta} = FFT(u_o)_y = U \sin(\omega t + \theta) 
\end{cases} \tag{7}$$

C. The Relationship between Space Vectors

$u_{\alpha\alpha}$ and $u_{\alpha\beta}$ are transformed into $u_{\alpha\alpha}$ and $u_{\alpha\beta}$ in $dq$ coordinate system by Park transformation. The relationship between the output voltage vector $\tilde{u}_o$ of the H bridge and the output voltage vector $\tilde{u}_i$ of the inverter is shown in Fig. 6. The black lines represent the stationary coordinate system, the green lines represent $dq$ coordinate system, and the blue lines represent the voltage vector.

![Fig. 6. The vector diagram of $dq$ coordinate system.](image)

In the figure, $u_{\alpha\alpha}$ and $u_{\alpha\beta}$ are the components of $\tilde{u}_o$ in $dq$ coordinate system. Similarly, $u_{\alpha\alpha}$ and $u_{\alpha\beta}$ are the components of $\tilde{u}_i$ in $a\beta$ coordinate system. $u_{\alpha\alpha}$ and $u_{\alpha\beta}$ are the components of $\tilde{u}_i$ in $dq$ coordinate system.

III. Design of Dual Closed-loop Controller for the Inverter

To reduce the complexity of the hardware and realize the output voltage track the desired voltage without static difference, Based on the voltage vector and virtual circuit, the voltage dual closed-loop control can be used. The voltage outer-loop is to control the voltage vector in $dq$ coordinate system, and the output voltage can track the desired value without deviation in steady state. The voltage inner-loop is to achieve approximate decoupling and improve the dynamic response by designing the virtual circuit.

A. The Design of Voltage Outer-loop

Assuming that $u_{o*}'$ is the input of meeting $u_o = u_o'$, the vector relationship is shown in Fig. 7. The red lines represent the desired voltage vectors $\tilde{u}_o'$ and $\tilde{u}_i'$. $\tilde{u}_o'$ is a synthetic vector by the three phase extension. $\tilde{u}_i'$ is also a synthetic vector on $\alpha\beta$-axis. If the $\alpha$-axis component $u_{\alpha\alpha}'$ of $\tilde{u}_o'$ is a modulated signal, the output of the single-phase inverter is $u_o'$. When the amplitude $U' + \Delta U'$ and the phase angle $\omega t + \Delta \theta'$ of $u_o'$ vary with the load, the closed-loop control can make $u_{\alpha\alpha}$ tend to $u_{\alpha\alpha}'$. The vector relationship is shown in Fig. 8. The red lines represent the desired voltage vector $\tilde{u}_o'$ and $\tilde{u}_i'$. The blue lines represent the transient voltage vector $\tilde{u}_o'$ and $\tilde{u}_i'$ in the dynamic process. The components of $\tilde{u}_i'$ in $dq$ coordinate system and the desired value are given to the PI so that $\tilde{u}_i'$ approaches $\tilde{u}_i'^*$. When the steady state is reached, $u_{\alpha\alpha}$ is equal to $u_{\alpha\alpha}'$ and $u_{\alpha\beta}$ is equal to $u_{\alpha\beta}'$.

![Fig. 7. The vector diagram of the open-loop control.](image)

![Fig. 8. The vector diagram of the closed-loop control.](image)
\[
\ddot{u}_m = L C \frac{d^2 \ddot{u}_a}{dt^2} + \frac{L}{R} \frac{d \ddot{u}_a}{dt} + \ddot{u}_a.
\] (8)

\[
\dot{u}_m \text{ and } \ddot{u}_m \text{ can be expressed as } \ddot{u}_m = u_{ad} + j \omega u_{aq} \text{ and } \ddot{u}_m = u_{ad} + j \omega u_{aq} \text{ in } dq \text{ coordinate system. Then (8) can be further transformed into (9) in } dq \text{ coordinate system.}
\]

\[
e^{j\omega t} \ddot{u}_m = L C \frac{d^2 e^{j\omega t} \ddot{u}_a}{dt^2} + \frac{L}{R} \frac{d e^{j\omega t} \ddot{u}_a}{dt} + e^{j\omega t} \ddot{u}_a
\] (9)

where \( e^{j\omega t} \) is the introduced rotating factor. The transfer function of (9) can also be written in the form of a matrix shown in (10), and the structure of the control object is shown in Fig. 9.

\[
\begin{bmatrix}
  u_{ad}(s) \\
  u_{aq}(s)
\end{bmatrix} = A \begin{bmatrix}
  i_d(s) \\
  i_q(s)
\end{bmatrix}
\] (10)

where \( A \) is

\[
\begin{bmatrix}
  L C s^2 + 1 - L C \omega^2 & -L C \omega \\
  -L C \omega & L C s^2 + 1 - L C \omega^2
\end{bmatrix}
\]
and \( B \) is

\[
\begin{bmatrix}
  L s - \omega L \\
  \omega L & L s
\end{bmatrix}.
\]

From (10) and Fig. 9, \( u_{ad} \), \( u_{aq} \), \( u_{ad} \), and \( u_{aq} \) have coupling relationships and \( i_d \) and \( i_q \) also have coupling relationships.

To achieve decoupling, a feedforward decoupling strategy is taken, but increasing the feedback of the output current needs current sensors and current detection circuit so that the hardware is complex. In these coupling items and the forward transfer function, it is seen that \( L \) determines the strength of these coupling items and the influence of the output current on the input voltage. The approximate decoupling can be achieved if \( L \) is greatly reduced. Thus, in the design of the outer-loop, these relationships can be neglected between voltage and current including \( 2 L C \omega \), \( \omega L \) and \( L s \). Instead, designing the inner-loop reduces the above effect. (10) can be simplified into

\[
\begin{bmatrix}
  u_{ad}(s) \\
  u_{aq}(s)
\end{bmatrix} = \begin{bmatrix}
  u_{ad}(s) \\
  u_{aq}(s)
\end{bmatrix}
\]

(11)

where \( \alpha \) is

\[
\begin{bmatrix}
  L C s^2 + 1 - L C \omega^2 & 0 \\
  0 & L C s^2 + 1 - L C \omega^2
\end{bmatrix}.
\]

Since the components of the voltage vector in \( dq \) coordinate system are the DC quantities, the general PI regulator can be used. The voltage control equation is expressed as

\[
\begin{align*}
\dot{u}_m &= (k_p + \frac{k_i}{s})(u_{ao}(s) - u_{ao}(s)) \\
\dot{u}_m &= (k_p + \frac{k_i}{s})(u_{ao}(s) - u_{ao}(s))
\end{align*}
\] (12)

The outer-loop control block diagram is shown in Fig. 10. The red solid line frame is the PI feedforward control section of the voltage outer-loop and the red dashed box is a conventional decoupling control.

![Fig. 10. The block diagram of the outer-loop control.](image)

**B. The Design of Voltage Inner-loop**

From Fig. 9, the coupling factor of \( u_{ad} \), \( u_{aq} \), and \( u_{ad} \), \( u_{aq} \) is \( \omega L \), and the coupling factor of \( i_d \) and \( i_q \) is \( \omega L \). Besides, the forward transfer function of \( u_{ad} \), \( u_{aq} \) affected by \( i_d \), \( i_q \) is \( L s \).

The value of inductance determines the strength of the coupling and the influence of \( i_d \) and \( i_q \) on the modulated voltage \( u_{ad} \).

When the load changes, the freewheeling effect of the inductor causes the capacitor current to fluctuate, so that the output voltage has a big fluctuation. In particular, when the load is a diode load, only the voltage outer-loop cannot correct the deviation of the output voltage in the half cycle. To reduce the coupling relationship and decrease the direct influence of \( dq \)-axis current, and improve the dynamic response speed of the output voltage, the inner-loop controller based on the virtual impedance is designed.

When the load suddenly disconnect, the output voltage becomes larger and its phase is behind the desired voltage at an angle of \( \theta \) due to the suppression of the inductor on the current. The vector relationships are shown in Fig. 11.

![Fig. 11. The vector diagram of the inner-loop control after the load is disconnected](image)
The controller is designed shown in Fig. 12. The red dashed box is the transfer function of the original system, and the red solid frame is the transfer function of the new system.

Fig. 12. The block diagram of the inner-loop control

As the load is unknown, the original system can be seen as a controlled voltage source. Its transfer function is expressed as

\[ U_{in}(s) = \left( L C s^2 + 1 \right) U_o(s) + L s I(s) . \]  
(13)

When the inner-loop is a P controller, combining the original system with the controller is seen as a new system. The open-loop transfer function of the new system is

\[ U_{in}(s) = \left( L C s^2 + 1 \right) U_o(s) + \bar{L} s I(s) \]  
(14)

where \( \bar{L} = L / (1 + K_{p3}) \) . \( K_{p3} \) is the inner-loop proportional coefficient.

From the perspective of the circuit, the equivalent virtual circuit is suitable for the low-frequency range far below the carrier frequency of the PWM signal. The harmonics near the fundamental frequency and below the fundamental frequency have a great influence on the output voltage waveform. The larger the \( \bar{L} \) is, the better the effect of suppressing these harmonics is, so \( K_{p3} \) cannot be too large.

After adding the P controller, the new system can be equivalent to be shown in Fig. 13. The filter inductance is reduced to \( 1/(1 + K_{p3}) \) of the original, so the voltage and current of the \( dq \)-axis are approximately decoupled. In addition, when the load changes, the effect of inductance on current is reduced to make the output voltage quickly track the desired voltage. In summary, the inner-loop control is to achieve the approximate decoupling and speed up the dynamic response.

Fig. 13. The equivalent structure of the new system after adding the inner-loop.

C. System Control Structure Block Diagram

The overall block diagram of the system control is shown in Fig. 14.

Fig. 14. The system control block diagram.

IV. SIMULATION AND EXPERIMENTAL RESULTS ANALYSIS

To test and verify the dual closed-loop control scheme of the single-phase inverter based on the extended and virtual circuit, the models developed above are implemented in both computer simulations and the experimental hardware. The results are given below.

A. Simulation of PLECS and Analysis of Results

5kVA single-phase inverter is simulated in the time domain with programs written in PLECS. The simulation parameters of the system are as follows. The output voltage is 220V/50Hz. The DC-link voltage is 311V. The inductance is 2.2mH. The capacitance is 50uF and the triangular carrier frequency is 8kHz. Besides, parameters of the outer-loop PI regulator are \( K_{p1} = 1.75 \), \( K_{i1} = 20 \), \( K_{p2} = 1.75 \) and \( K_{i2} = 20 \), and the parameter of the inner-loop PI regulator is \( K_{p3} = 10.34 \). The full load (\( R=10\Omega \)) and the diode rectifier load (a diode in series with a 10Ω resistor) for the single-phase inverter are verified, respectively.

The voltage and current dual closed-loop controller and voltage dual closed-loop controller are simulated for its response to a full step load and the step time is 1.025s. The results are shown in Fig. 15. It can be seen from Fig. 15 that both controllers exhibit a similar “notch” in the output voltage. Considering the steady state though, it can be clearly seen that the voltage and current dual closed-loop controller exhibits a prominent “AC following error” while the voltage dual closed-loop controller regulates this error to zero in less that 1/6 cycle of the AC voltage.
Fig. 15. Simulation waveform comparison of voltage and current dual closed-loop (Top) and voltage dual closed-loop (Lower).

Fig. 16 shows the output voltage and output current waveform of the inverter with the diode rectifier load. When the diode load is on, it is equivalent to the full load. When the diode load is off, the system is no-load. From the figure, the output voltage of the inverter can still track the desired voltage.

B. Experiment and Result Analysis

The system uses Field Programmable Gate Array, i.e., FPGA, as the core to achieve the vector control of full bridge inverter, and the internal IP core to achieve FFT. Using the proposed control strategy builds the experimental platform. The experimental parameters are shown in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC-link voltage $V_d$/V</td>
<td>48</td>
</tr>
<tr>
<td>AC filter capacitor $C$/uF</td>
<td>50</td>
</tr>
<tr>
<td>AC filter inductance $L$/mH</td>
<td>2.2</td>
</tr>
<tr>
<td>Load Resistance $R$/Ω</td>
<td>9</td>
</tr>
<tr>
<td>Switching frequency $f$/kHz</td>
<td>8</td>
</tr>
<tr>
<td>Output AC line voltage $U_o$/V</td>
<td>26</td>
</tr>
<tr>
<td>Output AC voltage frequency $f$/Hz</td>
<td>50</td>
</tr>
</tbody>
</table>

The experimental platform is shown in Fig.17.

Fig. 16. The output voltage and current waveform with the diode load.

Fig. 17. Experiment platform.

Fig. 18. No-load inverter output voltage waveform.

It can be seen from Fig. 18, Fig. 19, Fig. 20 and Fig. 21 that the experimental results are basically consistent with the simulation results. It indicates that the control strategy proposed in this paper can make the output voltage accurately track the desired voltage. When the load is a diode load, the system still responds very quickly.
Thus, its hardware only needs voltage sensors, the corresponding detected circuit and a simple over-current protected circuit. In other words, the voltage dual closed-loop control strategy can reduce the complexity of the hardware.

The system simulation and experimental results show that the control strategy proposed in this paper is effective. Through the dual closed-loop voltage control of the single-phase inverter, the output accuracy and response speed of the inverter are greatly improved.

**REFERENCES**


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