Synchronous Vibration Suppression of Magnetic Bearing Systems without Angular Sensors

Hongbo Sun, Student Member, IEEE, Dong Jiang, Senior Member, IEEE, and Jichang Yang

Abstract—Active Magnetic Bearing (AMB) levitates rotor by magnetic force without friction, and it can provide active control force to suppress vibration while rotating. Most of vibration suppressing methods need angular speed sensors to obtain rotating speed, but in many occasions, angular speed sensor is difficult to install or is difficult to guarantee reliability. This paper proposed a vibration suppressing strategy without angular speed sensor based on generalized integrator and frequency locked loop (GI-FLL) and phase shift generalized integrator (PSGI). GI-FLL and high-pass filter estimate frequency from control current, PSGI is applied to generate compensating signal. Firstly, model of AMB system expressed by transfer function is established and effect of centrifugal force is analyzed. Then, principle and process of vibration suppressing strategy is introduced. Influence of parameters are analyzed by root locus and bode diagram. Simulation results display the process of frequency estimation and performance of displacement. Experiments are carried on a test rig, results of simulations and experiments demonstrate the effectiveness of proposed vibration suppressing strategy.

Index Terms—Active Magnetic Bearing (AMB), vibration suppression, frequency estimation, phase shift generalized integrator (PSGI)

I. INTRODUCTION

In many occasions where required high rotating speed, such as high-speed motor and flywheel energy storage, Active Magnetic Bearing (AMB) has been used to support the rotating rotor to eliminate friction and active control. Vibration is a common problem in rotating machine, which is difficult to solve by traditional bearings. AMB can provide possibility of vibration suppression with its controllable stiffness.

As a result of processing error, the geometric center and gravity center of rotor cannot coincide, which will produce centrifugal force when rotor rotating. In AMB system, centrifugal force causes displacement of rotor off center, and the off-center signal causes current fluctuation in coil by sensor, controller and amplifier. Vibration of displacement and current are two main vibration in AMB system. Centrifugal force is main source of vibration, as a result, frequency of vibration in AMB is mainly rotating frequency.


In most of vibration suppressing methods, information of rotating speed is necessary. Speed signal is usually obtained by angular sensor. But in many occasions, it is difficult to install an angular sensor, or angular sensor is damaged easily. As a result, some methods are applied to estimate rotating speed. Wook-Jin Lee[10] applied phase locked loop to estimate rotating speed and applied it to vibration suppression. Qi Chen[11] proposed a frequency estimator to obtain rotating speed. Both of these two methods have to utilize information of the rotor in two degrees of freedom, and the symmetry of these two degrees of freedom affects the result of frequency estimation. This paper proposes a strategy to reduce synchronous vibration based on phase shift generalized integration (PSGI) and frequency locked loop (GI-FLL). The information of angular speed is obtained from control current by GI-FLL and high-pass filter from one degree of freedom instead of two, then PSGI is applied to generate compensating signal to suppress vibration in displacement.

This paper is arranged as follow. In part II, transfer function from centrifugal force to displacement is analyzed, and...
amplitude and phase of synchronous vibration are obtained. The analysis and process of GI-FLL and PSGI are also introduced in part II, the parameters are selected by analyzing root locus. Results of simulations are shown in part III. Experiment results carried on a test rig are shown in part IV. Simulation results and experiment results demonstrate the effectiveness of proposed vibration suppressing strategy. Conclusions are summarized in part V.

II. VIBRATION SUPPRESSION STRATEGY

In this section, model of AMB is established based on transfer function. Effect of centrifugal force is analyzed in part A and vibration suppressing strategy is introduced in part B. Method of Parameters selecting based on root locus is also introduced.

A. Model of AMB System and Unbalance Analysis

A typical eight-pole structure radial AMB is shown in Fig. 1. Two degrees of freedom are set in orthogonal to share load average relatively. When a rotor rotating levitated by AMBs, the centrifugal force will cause synchronous vibration in radial. The displacement sensor detects position of rotor and transform it to electric signal. The controller generates instruction by information of displacement and control algorithm. The amplifier is usually chosen as current controller combined with switching converter, which produces actual current according to instruction from displacement controller. The current in coils produces magnetic force to levitate the rotor. Proportional-Integral-Differential (PID) control algorithm is widely used in displacement controller. When applying PID control strategy, the transfer function of controller can be expressed as

$$G(s) = \frac{X(s)}{I(s)} = \frac{k_i}{m s^2 - k_c}$$  \hspace{1cm} (2)

When applying PID control strategy, the transfer function of controller can be expressed as

$$C(s) = k_p + \frac{k_i}{s} + k_{pD}$$  \hspace{1cm} (3)

The amplifier has low-pass feature, transfer function of which can be expressed as

$$G_i(s) = \frac{\omega}{s + \omega}$$  \hspace{1cm} (4)

The sensor is regarded as a linear, whose gain is \( k_b \). Unbalance mass equivalent to the centrifugal force operating on the rotor. When the centrifugal force operates on the rotor, the control system can be expressed as Fig. 3.

![Fig. 3. Structure of control system with centrifugal force](image)

To simplify the expression, the amplifier is regarded as a linear temporarily. Transfer function from centrifugal force to displacement is

$$\frac{X(s)}{F(s)} = \frac{G(s)/k_i}{1 + k_b C(s) G(s)}$$  \hspace{1cm} (5)

Setting \( s^2 = j\omega \), and calculating the amplitude and phase of transfer function

$$\frac{X(j\omega)}{F(j\omega)} = \frac{\omega}{\sqrt{k_i^2 k_b^2 + k_c^2 + k_b^2 k_c^2 \omega^2}}$$  \hspace{1cm} (6)

$$\angle \frac{X(j\omega)}{F(j\omega)} = \tan^{-1} \frac{k_i k_c - k_b^2 k_c}{k_i k_b k_c - k_b^2 k_c \omega^2}$$  \hspace{1cm} (7)

In steady state, the form of centrifugal force is \( F(t) = e \omega \cos(\omega t + \theta_0) \). Where \( e \) is eccentricity, \( \omega \) is rotating angular speed, and \( \theta_0 \) is initial value of phase of centrifugal force. As a result, the amplitude of synchronous vibration in displacement is

$$X_s = e \omega \sqrt{k_i^2 k_b^2 - k_c^2 + k_b^2 k_c^2 \omega^2}$$  \hspace{1cm} (8)

According to the expression of amplitude, in magnetic bearing applied PID control strategy with constant control parameters, enlarging the \( k_b \) can help reduce vibration. However, enlarging \( k_b \) also increases gain of noise, which will cause unstable system. As a result, to reduce vibration, we can enlarge sampling gain of synchronous component merely. A phase shift generalized integrator (PSGI) segment has infinite gain at its resonant frequency. So when PSGI segment is
applied in control loop of displacement, vibration in displacement can be suppressed. The GI-FLL is applied to obtain the rotating frequency from control current. However, GI-FLL is sensitive to DC component in control current, so a high-pass filter is applied before GI-FLL. The AMB control system applied vibration suppressing strategy is shown in Fig. 4.

![Fig. 4. Structure of AMB system applied vibration suppressing strategy](image)

**B. Analysis of AMB System Applied GI-FLL and PSGI**

Frequency estimation segment is composed of high-pass filter (HPF) and Generalized integrator-frequency locked loop (GI-FLL). GI-FLL has been applied in power grid to estimate the frequency and shows pretty effect [12]. In AMB system, control current in coil is nearly sinusoidal with DC component. But GI-FLL is sensitivity to DC signal of input signal, DC component may cause failure of frequency estimation. As a result, a high-pass segment applied to filter out DC component is added in front of GI-FLL. The structure of frequency estimation segment is shown in Fig. 5.

![Fig. 5. Structure of frequency estimation segment](image)

The HPF is chosen as first-order segment and its transfer function is

\[ H_{HPF}(s) = \frac{s}{s + \omega_H} \]  

(10)

The transfer function of error \( V_e \) in frequency estimation segment can be expressed as

\[ E(s) = \frac{V_e(s)}{V_e(s)} = H_{F1}(s) \frac{s^2 + \omega^2}{s^2 + k_g \Omega s + \Omega^2} \]  

(11)

\[ Q(s) = \frac{V_e(s)}{V_e(s)} = H_{F1}(s) \frac{k_\Omega}{s^2 + k_g \Omega s + \Omega^2} \]  

(12)

The parameter \( \Omega \) is set as 200πrad/s. Drawing the bode diagrams of above transfer functions as Fig. 6. Assuming the frequency of input signal is \( \omega = 200\pi \text{rad/s} \), if the estimated frequency \( \Omega \) is larger than \( \omega \), phases of \( V_e \) and \( V_b \) are the same, product of \( V_e \) and \( V_b \) is positive, the negative gain \( k_R \) and integrator \( S \), will decrease \( \Omega \). If \( \Omega \) is smaller than \( \omega \), phases of \( V_e \) and \( V_b \) are positive, product of \( V_e \) and \( V_b \) is negative, the negative gain \( k_R \) and integrator \( S \), will increase \( \Omega \). This process of adjustment stops until \( \Omega \) equals to \( \omega \), which means frequency of input signal is observed. DC component of input signal causes DC component in \( V_e \) and \( V_b \), it will destroy the adjustment process based on sign of product of \( V_e \) and \( V_b \). There is DC component in control current, so HPF is necessary for frequency estimation by control current. Details for parameter design is introduced in paper [12].

![Fig. 6. Bode diagram of frequency estimation segment](image)

To generate compensating signal, a phase shift generalized integrator (PSGI) is applied, whose structure is shown in Fig. 7.

![Fig. 7. Structure of PSGI](image)

The transfer function of PSGI is

\[ N(s) = \left( k_{PN} + \frac{k_{IN}}{s} \right) \frac{s}{s^2 + \Omega^2} \]  

(13)

There is infinite gain at \( s = j\Omega \) of PSGI, and for common integrator \( 1/s \), there is infinite gain at \( s = 0 \). The proportional-integral (PI) segment contributes to shift phase. If the parameter are designed as \( k_{PN} = \varepsilon \cos \phi \), \( k_{IN} = \varepsilon \Omega \sin \phi \), the PI segment generates an phase phase lag \( \phi \), and gain \( \varepsilon \) when \( s = j\Omega \). The resonant frequency \( \Omega \) is set as 200πrad/s and \( \varepsilon = 1 \), bode diagram of PSGI with various \( \phi \) is shown in Fig. 8.

![Fig. 8. Bode diagram of PSGI with various \( \phi \)](image)

The bode diagram shows that all the PSGIs have infinite gain at \( s = j\Omega \) and various \( \phi \) generates different phase shift at \( s = j\Omega \). When PSGI is applied to the AMB control system, the open-
The loop transfer function of the control system is
\[
H(s) = \left[ C(s) + N(s) \right] G_i(s) G(s) k_x
\]
\[
= \left[ \frac{k_p s + k_i + k_{pu} s^2}{s} + \left( \frac{k_{PI}}{s} \right) \right] \frac{s}{s^2 + \Omega^2} \frac{\omega_p}{s + \omega_p} \frac{k_m}{m s^2 - k_x}
\]
\[(14)\]

The characteristic equation of system is \( H(s) = -1 \), which can be transformed as
\[
e \cos \varphi + \Omega \sin \varphi
\]
\[
\left( s^2 + \Omega^2 \right) \frac{k_p s + k_i + k_{pu} s^2}{s} + \frac{m s^2 - k_x}{s + \omega_p} \frac{s + \omega_p}{s} \frac{k_m}{m s^2 - k_x}
\]
\[
= -1
\]
\[(15)\]

The root locus starts at \( s = \pm j \Omega \) and zeros of \( Z(s) \), where
\[
Z(s) = \frac{k_p s + k_i + k_{pu} s^2}{s} + \frac{m s^2 - k_x}{s + \omega_p} \frac{s + \omega_p}{s} \frac{k_m}{m s^2 - k_x}
\]
\[(16)\]

Because of \( s = \pm j \Omega \) locating at virtual axis, to make the system stable, the departure angle \( \Theta \) at \( s = \pm j \Omega \) should meet \( \pi/2 < \Theta < 3\pi/2 \). The relationship between stability and departure angle \( \Theta \) can be expressed as Fig. 9.

Calculating the departure angle as
\[
\theta = \pi - \varphi - \arg (Z(j\Omega))
\]
\[(17)\]

In order to select the proper \( \varphi \), the phase-frequency characteristics of \( Z(s) \) is obtained as Fig. 10.

To make the system stable, the parameter \( \varphi \) in different rotating frequency is chosen as TABLE I.

The parameter \( \varepsilon \) is designed by root locus. Drawing the root locus based on the characteristic equation as Fig. 11. The magnitude of \( \varphi \) in various rotating frequency is shown in Table I.

<table>
<thead>
<tr>
<th>Rotating Frequency</th>
<th>Magnitude of ( \varphi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(30Hz, 50Hz)</td>
<td>-45°</td>
</tr>
<tr>
<td>(50Hz, 100Hz)</td>
<td>-90°</td>
</tr>
<tr>
<td>(100Hz, 160Hz)</td>
<td>-132°</td>
</tr>
<tr>
<td>&gt;160Hz</td>
<td>-153°</td>
</tr>
</tbody>
</table>

When \( \varepsilon \) is small, the dominant poles are mainly determined by locus \( L_1 \). Larger \( \varepsilon \) makes the root locus move farther from virtual axis, which makes the system convergent faster. Larger \( \varepsilon \) also enlarges effect of locus \( L_2 \). When \( \varepsilon \) is larger than \( \varepsilon_r \), the root locus expand to the right of virtual axis, which means the system is unstable. In simulation or experiment, a look-up table can be made to select proper \( \varepsilon \) in various rotating speed.

When PSGI is applied in AMB system, the transfer function from centrifugal force to displacement can be derived as equation (18).

The frequency response of centrifugal force in displacement is shown in Fig. 12.

To make the system stable, the parameter \( \varphi \) in different rotating frequency is chosen as TABLE I.

Establishing the simulation model in Matlab/Simulink based on the transfer function of AMB system. The parameters are shown in TABLE II.
Displacement of rotor under influence of proposed vibration suppressing strategy is shown in part A, and part B presents transition process of control current and frequency estimation, which demonstrates frequency estimation is also effective when vibration reduces.

A. Performance of Rotor while Vibration Suppressing

The rotating speeds is set as 100Hz, at 0.3s, the high-pass filter and GI-FLL is applied to estimate the frequency by control current. At 0.8s, the PSGI is applied to reduce vibration. Fig. 13 presents results of displacement of rotor. After applying vibration suppressing strategy, the displacement of rotor is reduced significantly.

B. Performance of GI-FLL and Control Current while Vibration Suppressing

When PSGI is applied to the control system, there is a transition process in both current and displacement. The GI-FLL should work efficiently in this process. Fig. 14 presents the result of control current and Fig. 15 presents the result of frequency estimation by control current. The initial value of frequency estimation is 180Hz. At 0.3s, frequency estimation is applied. At about 0.5s, there is little error between actual frequency and estimated frequency. At 0.8s, PSGI is applied, there is a transition process in control current, the GI-FLL can also estimate the rotating frequency.

IV. EXPERIMENTAL RESULTS

To verify the effectiveness of the vibration suppressing strategy, experiments have been carried out on an AMB test rig. There are four degrees of freedom in radial controlled by two radial AMB. The shaft is driven by a high-speed motor. Two touchdown bearings are applied to protect the AMBs. Fig. 16 shows the components and arrangement of the test rig.

The structure of amplifier is shown in Fig. 17, the control board receives displacement with current signal and generate PWM signal by control algorithm. The switching device is IGBT and the switching frequency is set as 20kHz. The DC bus voltage is 150V and the bias current of AMB is 5A.

Parameters used in experiment is shown in TABLE III. Because of modeling error, parameters of PSGI is different from simulation. Other parameters are same as TABLE II.

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>PARAMETERS OF SIMULATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol</td>
<td>Quantity</td>
</tr>
<tr>
<td>m</td>
<td>Mass of rotor</td>
</tr>
<tr>
<td>k_x</td>
<td>Stiffness</td>
</tr>
<tr>
<td>k_i</td>
<td>Stiffness</td>
</tr>
<tr>
<td>ε</td>
<td>Eccentricity</td>
</tr>
<tr>
<td>ω_c</td>
<td>Bandwidth of amplifier</td>
</tr>
<tr>
<td>ε_c</td>
<td>Gain of PSGI</td>
</tr>
<tr>
<td>k_g</td>
<td>Gain of GI</td>
</tr>
<tr>
<td>Γ</td>
<td>Gain of PLL</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE III</th>
<th>PARAMETERS OF EXPERIMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol</td>
<td>Quantity</td>
</tr>
<tr>
<td>ε</td>
<td>Gain of PSGI</td>
</tr>
<tr>
<td>φ</td>
<td>Phase shift</td>
</tr>
<tr>
<td>k_p</td>
<td>Proportion</td>
</tr>
<tr>
<td>k_i</td>
<td>Integration</td>
</tr>
<tr>
<td>k_d</td>
<td>Differentiation</td>
</tr>
</tbody>
</table>
A. Displacement of Rotor while Vibration Suppressing at Constant Rotating Frequency

The displacement of rotor is shown in Fig. 18 and whose spectrum is shown in Fig. 19. The rotating speed is set as 80Hz, before the vibration suppressing strategy is applied, the amplitude of synchronous vibration is about 80.49μm. After applying suppressing strategy, the synchronous vibration reduces to 2.06μm, which shows the effectiveness of the suppressing strategy.

![Fig. 18. Displacement of rotor without and with proposed strategy](image1)

![Fig. 19. Spectrums of displacement without proposed strategy](image2)

B. Performance of GI-FLL and Control Current while Vibration Suppressing at Constant Rotating Frequency

Transition process of control current is shown in Fig. 20. The DC component in control current is about 3A. Result of frequency estimation is shown in Fig. 21. The error between actual frequency and estimated frequency is about ±0.5Hz before PSGI is applied. After applying PSGI, the frequency error increases to about ±2Hz, but this error hardly influences effect of vibration suppressing. This error is mainly caused by two factors: firstly, there is noise in current signal. Then, when PSGI is applied, synchronous vibration of control current is also reduced, the frequency estimation mainly utilizes the synchronous vibration in control current, so result of frequency estimation is also affected by harmonic vibration.

![Fig. 20. Performance of control current when PSGI is applied](image3)

![Fig. 21. Frequency Estimation when PSGI is applied](image4)

C. Performance of Rotor while Vibration Suppressing at Variable Rotating Frequency

Accelerating rotating frequency from 50Hz to 70Hz. Result of displacement is shown in Fig. 22. In 0-12s, the rotor rotates with proposed vibration suppression strategy, the amplitude of vibration is about ±5μm. In 12-22s, rotating frequency accelerates from 50-70Hz, and during acceleration, GI-FLL and PSGI are applied at the same time. At 40s, displacement reduces to ±5μm. When the rotor rotates at 50Hz and 70Hz without vibration suppression, the amplitude of vibration is larger than 50μm. This result presents that after transition process caused by acceleration, the proposed strategy also works effectively. However, the frequency estimation should be improved to adopt accelerating.

![Fig. 22. Displacement while accelerating](image5)
Transition process of control current while accelerating is shown in Fig. 23. Results of frequency estimation is shown in Fig. 24. The vibration in current while accelerating increases but reduces after acceleration. The result of frequency estimation shows that GI-FLL can estimate the rotating frequency while accelerating to support the PSGI for vibration suppression.

![Fig. 23. Performance of control current while accelerating](image1)

![Fig. 24. Frequency estimation while accelerating](image2)

V. CONCLUSIONS

This paper analyzed performance of rotor by influenced of centrifugal force in AMB system. The amplitude and phase of synchronous vibration in displacement is calculated in theory. Considering that angular speed sensor is necessary in most of vibration reduction methods, this paper proposed a vibration suppression method without angular speed sensor based on GI-FLL and PSGI. GI-FLL is applied to obtain rotating speed by control current and HPF is used to filter out DC component in control current. PSGI generates compensating signal to suppress vibration. The value of compensating phase $\phi$ and the effect of parameter $\varepsilon$ is analyzed by root locus. Results of simulations and experiments demonstrate the effectiveness of vibration reduction effect of proposed vibration suppressing strategy in constant rotating speed. The robustness for frequency estimating while speed changing will be further improved. Then, the stability analysis of proposed control strategy is carried out for the single degree of freedom system in this paper. However, the four degrees of freedom of radial motion are coupled in AMB-rotor systems. Thorough analysis of stability considering coupling effect and nonlinear effect will be further researched.

REFERENCES


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